Vortex rings from cylinders with inclined exits

D. R. Webster and E. K. Longmire

Department of Aerospace Engineering and Mechanics, University of Minnesota, Minneapolis, Minnesota 55455

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A typical experimental vortex generator was perturbed by inclining the exit orifice. Instantaneous velocity fields were measured with particle image velocimetry at a Reynolds number, \( \Gamma_0/n \), of 2800, which falls in the laminar regime for the axisymmetric case. Despite the nearly uniform velocity of the axisymmetric piston, the velocity exiting the cylinder is spatially and temporally non-uniform. Specifically, the exit velocity and the entrainment are larger on the short cylinder side. This fluid motion leads to an initial vortex roll-up with maximum and minimum circulation at the shortest and longest cylinder locations, respectively. A highly complex vortex structure forms, consisting of a primary vortex ring with varying circulation and branched vortex tubes that initially extend from the primary ring upstream toward the cylinder. The variation of the circulation in the primary ring and the strength of the branched vortex tubes increase with incline angle. The branched vortex tubes induce a strong cross-stream sweep of fluid toward the long cylinder side. The branched tubes convect across the cylinder exit with the sweep, break free of the cylinder, and pass through the primary ring. Beyond this time, the vortex structure consists of two closed-loop branches connected on the short cylinder side. As the flow progresses, the center of momentum moves toward the short cylinder side. As the cylinder incline angle is increased, the migration away from the centerline increases, and the flow becomes increasingly disorganized. The propagation speed and penetration distance are reduced because of the loss of coherent circulation. Qualitatively similar velocity fields and flow visualization photographs are presented for a larger (nominally turbulent) Reynolds number of 23000. © 1998 American Institute of Physics. [S1070-6631(98)00902-7]

I. INTRODUCTION

Vortex rings have been studied extensively due to their fundamental importance in fluid dynamics and turbulence mechanics. In fluids with constant density, individual rings are usually generated by imparting an impulse to a finite volume of fluid. The total impulse determines the circulation and propagation speed of the ring. In terms of practical applications, axisymmetric vortex rings are interesting because they are efficient at transferring coherent packets of momentum across relatively long distances.

In the laboratory, an axisymmetric vortex ring is typically generated by displacing a volume of fluid through an orifice or cylinder. The flow separates from the orifice edge forming a cylindrical vortex sheet that subsequently rolls up into a vortex ring. As the ring propagates away from the orifice, it entrains irrotational ambient fluid and sheds packets of vortical fluid into the wake. The shedding of vortical fluid decreases the core circulation and consequently decreases the propagation speed. While the characteristics of axisymmetric vortex rings are fairly well understood, rings that are initially non-axisymmetric have received limited attention. This study examines the consequences of perturbing vortex ring symmetry by inclining the cylinder exit.

Trailing edge geometry has a profound effect on the vortex dynamics in other shear flows. For example, in round jets with stepped, sawtooth, and crown nozzles, discontinuities in nozzle edge location or slope introduce streamwise vortices which significantly enhance mixing with the ambient environment. In contrast, streamwise vortices do not form in jets exiting inclined nozzles because they have continuous edge location and slope.

A common behavior observed in jets from inclined nozzles is the formation of inclined vortex rings with slightly less incline than the nozzle edge. As these rings propagate downstream, they migrate away from the centerline and become increasingly inclined until three dimensional breakdown occurs. The downstream evolution and interaction of the inclined vortex rings is a function of several parameters including roll-up frequency and shear layer thickness. Under some circumstances, vortex rings form normal to the centerline of the jet. In all cases, the inclined nozzles enhance the overall mixing and radial growth of the jet. Planar mixing layers with a skewed trailing edge reveal similar behavior. Depending on the roll-up frequency, the vortex tubes are aligned parallel to the skewed edge or normal to the mean flow direction.

A number of studies have examined three-dimensional aspects of vortex rings that were initially axisymmetric. One area of interest has been the growth of azimuthal instability waves. Weigand and Gharib recently provided some illustrative flow visualization of the transition from an axisymmetric laminar ring to a turbulent ring through the growth of azimuthal waves. The onset of turbulence was...
associated with intermittent shedding of core vorticity and an increase in the decay rate of the propagation velocity. The turbulent ring appeared to maintain a stable core as the total circulation decreased and eventually relaminarized. Thus, even a turbulent ring could maintain a very stable propagation path.

There are few examples of experimental vortex ring generators with non-axisymmetric orifices. Dhanak and de Bernardinis\textsuperscript{16} simulated and performed flow visualization of vortex rings generated from elliptical orifices. As the elliptical rings evolved, the orientation of the major and minor axes switched periodically, and the rings bowed relative to the orifice plane. Ellipticity increased the propagation speed, but the effect of ellipticity on vortex breakdown is unknown. For an extreme ratio between the minor and major axis $b/a \approx 0.2$, the elliptic vortex ring pinched off near the center, thus forming two closed rings each with reduced ellipticity. Recent numerical simulations confirm that elliptic vortex rings and rings subjected to strain or shear fields tend to pinch off.\textsuperscript{17,18} Elliptical vortex rings formed in the shear layer of a jet of elliptical cross section show similar behavior including axis switching, bowing, and pinching off.\textsuperscript{19,20}

While it is clear that trailing edge geometry is a strong perturbation in other shear flows, its effects on vortex rings are unknown. The goal of the current study was therefore to examine the effect of a simple geometric perturbation on vortex ring formation and propagation. The flow was perturbed by inclining the cylinder exit relative to the centerline axis of a round generator. The effect of cylinder incline angle, piston geometry, and Reynolds number were examined.

II. FACILITY AND EXPERIMENTAL TECHNIQUES

The experiments were conducted in a glass tank of square cross section with 400 mm sides and 760 mm depth. The working fluid was water. Vortex rings were generated by displacing a piston at nearly constant speed through a submerged cylinder. The piston driver mechanism is shown in Fig. 1. A gage pressure of 1.6 atm was applied to a hydraulic cylinder by opening a high-speed poppet solenoid valve. The activated cylinder drove an aluminum plate downward a distance of 45.0 mm before it contacted and displaced an impact plate. The plates moved together at a nearly constant velocity ($U_0 = 77.2$ mm/s or 628 mm/s) dictated by an adjustable viscous damper. The piston, which was attached to and driven by the impact plate, displaced a volume of fluid from a cylindrical housing. A fixed aluminum block impulsively stopped the motion. An average of 10 measurements of the slower piston velocity-time record is shown in Fig. 2. For a given event, the velocity program factor defined by Glezer was less than 1.01, where a value of 1.00 corresponds to an ideal top hat velocity profile.\textsuperscript{21}

The submerged cylinders were formed from stainless steel pipe with an inner diameter (D) of 72.8 mm and a wall thickness of 1.7 mm. Three cylinders with inclined exits and a flat (axisymmetric) reference cylinder were tested. The exits of the inclined cylinders were cut along planes such that the axial distances between the longest and shortest locations were D/4 (14°), D/2 (26°), and D (45°). Unless otherwise noted the polyethylene piston had an axisymmetric flat face. The axisymmetric piston was chosen in order to minimize radial fluid motion within the cylinder. In all cases, the piston stroke length, $L$, was equal to D. For the low Reynolds number results presented, the final piston position was located a distance D upstream of the average cylinder lip location. Other piston positions and piston geometries were tested, and the results are reported in Section III C.

The results are described using a cylindrical coordinate system ($r$, $\psi$, $x$) that originates along the centerline at the average cylinder lip location. The angle $\psi$ is defined to be zero at the location where the cylinder lip was longest and $\pi$ where it was shortest. The time scale is defined by the piston velocity and stroke length, $t^*=(t-t_0)U_0/L$, where $t_0$ was the time that the piston started moving. Thus, the piston moved from $t^*=0$ until 1.

Flow visualization was performed by filling the tank with deionized water and 0.01 bromothymol blue by mass. The ambient color was brown. The fluid within the cylinder
was darkened by applying a 15 volt potential between the cylinder housing and a thin metal plate on the piston face for 15 minutes. The darkened fluid was then displaced by the piston motion. The flow visualization tests were captured on S-VHS format tape with a Panasonic video camera. The images discussed below are selected from sequences that appear in a companion paper.22

Instantaneous velocity fields were measured using particle image velocimetry (PIV). Illumination sheets were generated with two pulsed Nd:YAG lasers arranged side-by-side. The beam paths were combined with a polarization beam splitter such that the on-axis beam was transmitted by the “splitter” while the off-axis beam was reflected. Each laser pulsed at 10 Hz with a pulse duration of 7 ns. The delay between pulses was 5.0 ms and 0.75 ms for the slow and fast piston velocity, respectively. The energy output per pulse was approximately 100 mJ. A 1 m focal length spherical lens and a 19 mm focal length cylindrical lens were used to create each illumination sheet. The thickness of each sheet was approximately 1 mm throughout the photographed flow field. The water was seeded with TiO$_2$ particles with a specific gravity of 3.5 and nominal diameter of 3 μm. The inertial time constant of the particles was approximately 2 μs, and the settling velocity was 12 μm/s, both of which were negligibly small for this flow.

Single images of double-pulse events were acquired with a Kodak DCS 420M digital camera and a Nikon AF Micro-Nikkor lens with 60 mm focal length. The monochrome pixel array size was 1012×1524, and each pixel depth was 10 bits. The ISO equivalent was 1000, and the aperture setting was f8. The camera and a rotating mirror were located in a plane parallel to the illumination sheet. The focal distance was 732 mm. The resulting resolution for the digital images was 8.3 pixels/mm in the illumination plane. The rotating mirror imposed an artificial shift on the image in order to resolve directional ambiguity and to alleviate the large velocity disparity between the displaced and ambient fluid. The mirror position was controlled with a Cambridge Technology coil galvanometer-based scanner (Model 6650).

In the case of the slow piston velocity, the rotation rate of the

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**FIG. 3.** Velocity field for the flat cylinder at (a) $t^* = 1.56$, (b) 2.62, (c) 3.68, (d) 4.74, (e) 5.80, and (f) 6.86. Contour line in (b) denotes vortex core boundaries. $Re=2800$. 

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mirror was 0.14 rad/s. Thus, the artificial shift corresponded to 8.55 pixels (or a velocity of 205 mm/s). For the high Reynolds number case, the shift was 9.25 pixels or 1450 mm/s. For Figs. 10 and 11, a Nikon AF Micro-Nikkor lens with 105 mm focal length was used at the same focal distance. The resulting resolution was 14.6 pixels/mm in the illumination plane. The mirror rotation rate was 0.105 rad/s corresponding to a shift of 11.2 pixels or a velocity of 154 mm/s.

Timing and control signals were generated with a Macintosh computer and National Instruments NB-MIO-16X (12bit DAC) board. Separate digital-to-analog converter channels were used to generate a square wave for the external timing circuit and a sawtooth wave for the rotating mirror controller. The timing circuit controlled the laser pulsing and camera shutter such that the camera recorded the images when the mirror was nominally at 45°. The timing circuit also triggered the poppet solenoid valve.

Acquired images were stored on a PCMCIA Type III card inside the camera and subsequently downloaded with Adobe Photoshop to a PC clone through a SCSI adapter. Commercial software from Fluid Flow Diagnostics Inc. was used to calculate the average pixel displacement of image pairs in each interrogation area via a spatial auto-correlation method. Interrogation areas were 64×64 pixels, corresponding to a spatial resolution of 7.7×7.7 mm (4.4×4.4 mm in Figs. 10 and 11). The overlap of the interrogation area was 50% so that a 31×47 vector field was calculated for each image file. The image resolution and the time delay between laser pulses were used to calculate the velocity at each point. No interpolated vectors were substituted into the velocity field plots. Missing velocity vectors were filled by interpolation before calculating derived quantities such as circulation, vorticity, vortex core boundary, and center of momentum. In each figure, the vector tail is at the data location, and a reference vector is given. The dominant contribution to uncertainty in the velocity measurements is the uncertainty in the displacement within a given interrogation area. For this study, that uncertainty was found to be ±0.1 pixel yielding an uncertainty in velocity of ±3% of \( U_0 \).

III. RESULTS

Two Reynolds numbers (\( \Gamma_0/\nu \), where \( \Gamma_0 = U_0^2/9 \) is the ideal circulation for the axisymmetric ring, \( T \) is the travel time of the piston, and \( U_0 \) is the constant piston velocity) were examined: 2800, which is in the laminar regime of Glezer’s transition map for L/D = 1, and 23000, which is in the turbulent regime. Flow development in the lower Reynolds number case is discussed in detail in Sections III A and III B. Effects of initial conditions and Reynolds number are discussed in Section III C. Additional aspects of the three-dimensional flow behavior including wake properties and dispersion are revealed by the flow visualization images presented in Section III D.

A. Formation of the vortex structure

The flow exiting the flat cylinder at \( \text{Re}=2800 \) is shown in Fig. 3. Fluid exits the cylinder uniformly, separates from the edge, and rolls up into an axisymmetric vortex ring. In Fig. 3(a), the piston has stopped moving, and the vortex ring has separated from the cylinder. At this time the ring diam-
eter is 1.25D, which is consistent with the ring diameter measured by Didden\textsuperscript{2} just after the piston stroke. Figure 4 shows the vorticity field for the flat cylinder case at \( t^* = 2.62 \). The vortex core is essentially round and appears uniform around the circumference of the ring. A weak image vortex caused by the piston stopping is observed near the cylinder exit.

In order to define the vortex core boundary, we have employed the method of Jeong and Hussain.\textsuperscript{23} The method requires the calculation of the eigenvalues of the \( S^2 + \Omega^2 \) tensor in order to identify local pressure minima due to vortical motion. A vortex core is defined as a continuously connected region with two negative eigenvalues. This region corresponds to the zone where the magnitude of the azimuthal velocity increases with radial distance from the core center. Hence, the core boundary shows the positions of maximum azimuthal velocity. In Fig. 3\textsuperscript{a}, we have plotted the zero contour of the second eigenvalue where the first eigenvalue is negative. The primary vortex core in the flat cylinder case is identified easily with this method, but the boundary is not perfectly round due to experimental uncertainty and data coarseness. From this plot, the vortex core diameter is estimated to be 0.18D, which agrees reasonably with Didden.\textsuperscript{2} The core of the image vortex is also identified near the cylinder exit.

The flow from the inclined cylinders is significantly more complex than in the axisymmetric case, even at very early times. Figure 5 shows a sequence of velocity fields during the initial formation of the vortex structure in the D/2 cylinder case. At \( t^* = 0.18 \), the velocity magnitude varies across the cylinder exit, with the largest magnitude occurring in the \( \psi = \pi \) half-plane. Also, the flow is directed radially outward in the \( \psi = \pi \) half-plane, while it is nearly axial in the \( \psi = 0 \) half-plane. Figure 6 illustrates the initial flow pattern. High pressure resulting from the piston motion drives the flow toward the relatively low pressure in the ambient fluid. Because of the inclined cylinder exit, the path lines are curved as shown. This motion, coupled with conservation of mass inside of the cylindrical boundary, results in larger initial velocity in the \( \psi = \pi \) half-plane. Even though the exit velocity is more uniform by \( t^* = 0.71 \), the initial non-uniformity causes a strong variation in the vortex core formation. For instance, at \( t^* = 0.71 \) the vortex core in the \( \psi = \pi \) half-plane is entraining ambient fluid and appears to be separated from the cylinder. In contrast, the vortex core in the \( \psi = 0 \) half-plane shows little or no entrainment. At \( t^* = 1.03 \), both cores are entraining ambient fluid, but the entrainment is much stronger in the \( \psi = \pi \) half-plane. By comparing these half-planes, it is evident that the flow varies significantly around the circumference.

Figure 7 shows the velocity field for the D/2 cylinder at \( t^* = 2.62 \). The flow differs strongly from the flat cylinder exit case at the same time (Fig. 3\textsuperscript{b}). In Fig. 7\textsuperscript{b}, the vortex core location and strength have reflective symmetry between the \( \psi = \pi/2 \) and \( 3\pi/2 \) half-planes. However, unlike the flat cylinder case, the flow converges strongly toward the centerline near the cylinder, and the axial velocity is large near the centerline at the axial location of the core. The \( \psi = 0 \) and \( \pi \) plane shows a radically different flow from the flat cylinder case. The vortex core is inclined at an angle less than the cylinder exit angle, and there is a powerful sweep of fluid into the \( \psi = 0 \) half-plane upstream of the core. A comparison of the two views shows that the converging vectors in the \( \psi = \pi/2 \) and \( 3\pi/2 \) plane are feeding fluid into the sweep which is focused near the \( \psi = 0 \) and \( \pi \) plane. Furthermore, the vortex core appears to have non-uniform circulation. In fact, the vortex core has significantly larger circulation in the \( \psi = \pi \)
half-plane than in any of the three other half-planes shown. This is a surprising observation since the Helmholtz vortex theorem states that the circulation of a vortex tube is constant along its length.

In addition to altering the circulation, the exit geometry affects the size and shape of the vortex core. Figure 8 shows the vorticity field for the D/2 cylinder at \( t^* = 2.62 \). The vorticity contours indicate that in the \( \psi = \pi \) half-plane, the core is larger than in the flat cylinder case (Fig. 4), while in the \( \psi = 0 \) half-plane, it is smaller. Furthermore, the shape of the region of positive vorticity in the \( \psi = \pi \) half-plane is distorted compared with the round core observed in the reference case. The core boundary shown in Fig. 7(a) confirms the relative core size and the distortion in the \( \psi = \pi \) half-plane. The core diameters are estimated to be 0.30D and 0.11D, respectively. The core diameter for the flat, D/4 (discussed below), and D/2 cases roughly follows the simple scaling, \( \delta \propto (\Gamma / U_0) \). In Fig. 7(a), several additional small regions are highlighted owing to the sensitivity of plotting the zero contour of the second eigenvalue evaluated from experimental data. Comparison with the vorticity field indicates that these are not significant vortex cores.

In order to understand the circumferential variation in circulation, the circulation of the primary vortex core was determined in eight circumferential half-planes for each cylinder exit angle. The circulation was calculated by performing the line integral of velocity around the core. (This method showed excellent agreement with calculations of the area integral of vorticity.) The results, which are plotted in Fig. 9, represent ensemble averages of the circulation calculated from three independently acquired velocity fields. The uncertainty of \( \Gamma / U_0 \) was estimated to be 0.02, based on the uncertainty in velocity, variation between flow events (repetitive velocity fields), and variation due to integration path.

The cylinder with the flat exit has uniform circulation of \( \Gamma / U_0 D = 0.64 \). The slug model for the rate of vorticity flux from the edge of the orifice predicts \( \Gamma / U_0 D = 0.5 \). Hence, the measured value is 28% greater, which is consistent with the observation by Glezer \( ^{21} \) that the circulation is typically 30%–40% larger than the slug model prediction. In distinct contrast to the flat case, the circulation varies around the circumference in each inclined case. For the D/2 exit case introduced above, the circulation is smallest (\( \Gamma / U_0 D = 0.34 \)) on the long side (\( \psi = 0 \)) and largest (0.83) on the short side (\( \psi = \pi \)). Furthermore, the variation depends on the incline angle: the circulation in the D/4 cylinder case deviates only mildly from the flat case, while the deviation in the D cylinder case is more extreme.

The circumferential variation in circulation is primarily a result of the non-uniform velocity exiting the cylinder. In particular, the larger initial exit velocity near \( \psi = \pi \) leads to larger circulation. The slug model states that the circulation is proportional to the square of the velocity at the orifice exit. In all inclined cases, the largest circulation (Fig. 9) and the largest initial exit velocity (Fig. 5(a), for example) are both in the \( \psi = \pi \) half-plane. Note that shear layer development length does not play an important role because it is uniform around the circumference. Specifically, the piston motion displaces the same ‘‘stroke length’’ of fluid regardless of circumferential location. Thus, all of the fluid near the cylinder wall moves through approximately the same distance during the stroke.

It is important to reconcile the Helmholtz vortex theorem with the circumferential variation of circulation in Fig. 9. The Helmholtz theorem states that the strength of a vortex tube (i.e., the circulation) is constant along its length. A corollary is that a vortex tube cannot end in the fluid, hence it must be connected to a solid boundary or form a closed loop. A simple vortex ring is the usual example of the closed loop case. As expected, the flat cylinder exit case forms a closed loop, uniform circulation ring. In the inclined exit case, however, the primary vortex ring does not have sufficient circulation at \( \psi = 0 \) to form a closed loop that is consistent with the Helmholtz vortex theorem. Hence, an important question is: what does the vortex tube with large circulation at \( \psi = \pi \) connect to? To help answer this question, a close-up view of
the D/2 case at $t^* = 2.62$ (same perspective as Fig. 7(b)) is shown in Fig. 10. The primary core used to calculate the circulation value in Fig. 9 is located near $r/D = 0.65$. Note the two additional cores present in each half-plane near the centerline. These additional cores do not appear in the flat cylinder exit case. In the D/4 cylinder case, shown in Fig. 11, a weaker single elongated core is observed in each half-plane near the centerline. In each case, the sum of the circulations in the inner cores and in the primary core agrees with the maximum circulation measured at $\psi = \pi$. In Fig. 10 the primary core has $\Gamma / U_0 D = 0.39$, and the branched cores in the $\psi = 3\pi/2$ half-plane have $\Gamma / U_0 D = 0.42$ for a sum of 0.81, which agrees with the $\psi = \pi$ half-plane circulation of 0.83 (maximum in Fig. 9). In Fig. 11 the primary and branched cores have $\Gamma / U_0 D = 0.64$ and 0.16, respectively, compared with the maximum of 0.76 in Fig. 9. Thus, the Helmholtz theorem appears to be satisfied if the branched cores are connected to the strong core at $\psi = \pi$.

From the evidence presented above and the knowledge gained by examining the flow field in eight circumferential half-planes, the three-dimensional circulation pattern can be reconstructed. A sketch of this structure is shown in Fig. 12. At this time, the flow contains a primary vortex ring with varying circulation and a pair of branched tubes near the centerline that extends upstream into the cylinder. For simplicity, Fig. 12 shows only one branched tube in each half-plane. Vorticity plots show that, for the D/2 cylinder case at $t^* = 2.62$, the branching location occurs near $\psi = 3\pi/4$.

The reconstructed vortex structure is consistent with the velocity field at $t^* = 2.62$ (Fig. 7). From Biot–Savart law considerations, the sweep of fluid into the $\psi = 0$ half-plane cannot be explained from the vorticity in the primary ring. It is, however, consistent with the induced motion from the branched vortex tubes. In the orthogonal view (Fig. 10(a)), the sweep appears as the relatively large axial velocity near the centerline. Again, this is consistent with induced motion from the branched vortex tubes.

FIG. 12. Sketch of the vortex structure at early time.
In considering why and how the branched tubes form in this flow, it is useful to reconsider the velocity field plots in Fig. 5. The exit velocity imbalance in Fig. 5a initiates a circulation imbalance. Because of the local velocity deficit, the core forming at $\psi=0$ cannot accumulate enough circulation during the piston stroke to match the strong circulation at $\psi=\pi$. By the end of the stroke, the core at $\psi=0$ detaches from the cylinder necessitating some additional vortical structure that must connect to the core at $\psi=\pi$. Furthermore, because the primary core detaches first at $\psi=\pi$ (see Fig. 5b), we can assume that the additional structure is initiated at or close to that location. By $t^*=1.03$ (Fig. 5c), the additional structure (the branched tubes) has already begun to induce the cross stream sweep upstream of the core.

The initial vortex structure is quite different from that observed in a round jet exiting an inclined nozzle. In the jet, the vortex rings are simple, uniform circulation, inclined loops that form nearly parallel to the nozzle lip. There is no evidence to suggest a complex vortex structure analogous to the observations here. In contrast to the initial velocity distribution in the impulsively generated flow, the velocity of the round jet was continuous and uniform across the nozzle exit. This supports the conclusion that the complex vortex structure observed in the current work results from the unique fluid motion exiting the cylinder.

**B. Evolution and propagation of the vortex structure**

The evolution of the axisymmetric vortex ring is shown in Fig. 3. After separating from the cylinder, the ring propagates at 24.8 mm/s ($0.32U_0$) along the centerline and maintains its axisymmetric form. The non-dimensional propagation speed is $2\pi U_0 D/T = 3.1$, which is slightly lower than the value of 3.49 quoted by Weigand and Gharib for a laminar ring. The ring diameter decreases slightly after the piston motion stops and remains essentially constant at 1.14D in the measurement region. This result agrees very well with Didden for $L/D = 1.0$. The propagation continues beyond the...
The flow evolution from the D/4 cylinder, shown in Fig. 17, is qualitatively similar to the D/2 case. A milder sweep into the \( \psi = 0 \) half-plane upstream of the vortex cores is observed in Fig. 17(b). Since the circulation variation is milder in this case, the branched vortex tubes are weaker, which is consistent with the reduced sweep strength. The sweep collides with the vortex core in the \( \psi = 0 \) half-plane in Figs. 17(c) and 17(d), and the branched vortex tubes detach from the cylinder at a slightly later time than the D/2 case. By \( t^* = 5.80 \) (Fig. 17(e)), the primary ring is inclined opposite to the cylinder inclination. The relatively weak core corresponding to the branched vortex tube is observed down-
stream of the primary vortex core in the $\psi=0$ half-plane in both Figs. 17(e) and 17(f). Again, the overall migration of the flow is into the $\psi=\pi$ half-plane.

The vorticity field for the D/4 cylinder case (Fig. 18) indicates that the core diameter is larger and smaller in the $\psi=\pi$ and 0 half-planes, respectively, when compared to the axisymmetric reference flow. This is the same dependence observed in the D/2 case discussed above. An additional region of positive vorticity is observed in the $\psi=0$ half-plane. The velocity field (Fig. 17(b)) reveals that this positive vorticity results from the confluence of the cross-stream sweep and the upstream entrainment of the primary vortex core. Figure 17(c) indicates that this weak core remains near the cylinder while the primary structure propagates away. Figure 17(b) also shows the boundary of the vortex core. The vortex core has diameters of approximately 0.25D and 0.14D in the $\psi=\pi$ and 0 half-planes, respectively. This agrees with the relative size observed in the vorticity fields.

The evolution is similar, but more extreme, for the D
cylinder exit case shown in Fig. 19. At early time ($t^* = 0.50$), the velocity across the cylinder exit is extremely non-uniform; the exit velocity is much larger (by roughly a factor of 3) in the $\psi = \pi$ half-plane than in the $\psi = 0$ half-plane. The variation of exit velocity contributes to the strong circumferential variation of circulation as discussed in the previous section. The large difference in circulation between the cores in the $\psi = 0$ and $\pi$ half-planes is observed in Fig. 19 and quantified in Fig. 9. A powerful sweep of fluid into the $\psi = 0$ half-plane is observed in Figs. 19(b) and 19(c) near the inclined cylinder exit. Due to the large sweep velocity, the detachment of the branched vortex tubes from the cylinder is slightly earlier than in the D/2 case. The velocity field in Fig. 19(e) is very disorganized, but appears similar to the later stages of the D/2 case (Fig. 13(f)). The severe non-uniformity of the primary vortex ring and the powerful nature of the sweep (and branched vortex tubes) lead to a rapid breakdown of the coherent vortex structure. The breakdown inhibits the self-inductive motion, and the propagation of fluid away from the cylinder is clearly limited compared with the other cases.

Figure 20 shows the vorticity field at $t^* = 2.62$ for the D cylinder. The vorticity contours and core boundary shown in Fig. 19(c) reveal the same trend in core size and distortion observed in previous cases. The core shape in the $\psi = \pi$ half-plane is particularly different from the reference case. In fact, the core has broken apart into several distinct patches, an indication of the disorganized nature of the flow as observed in the velocity fields.

It is clear from the vector field plots that inclining the cylinder exit causes the overall momentum associated with the vortex structure to move into the $\psi = \pi$ half-plane. Figure 21 shows the trajectory of the center of momentum for each case. The symbols indicate the center of total momentum calculated from the velocity field in the $\psi = 0$ and $\pi$ plane. The center was calculated by integrating the momentum magnitude over the measured flow field, and the plotted data are ensemble averages of three independently measured.

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**FIG. 19.** Velocity field for the D cylinder at (a) $t^* = 0.50$, (b) 1.56, (c) 2.62, (d) 3.68, and (e) 4.74. Contour line in (c) denotes vortex core boundaries. Re = 2800.
The uncertainty of a given measurement is estimated to be ±0.05D. Note that for early times, some momentum remains within the cylinder, and thus it is not included in the measured values.

The trajectory of the flat cylinder case is along the center axis, as expected. The propagation speed of the center of momentum over this range is 23.3 mm/s, which is slightly slower than the estimate resulting from tracking the vortex core. This is consistent with the concept of minimal wake shedding over this axial range. In each of the inclined cylinder exit cases, the center of momentum begins in the $\psi=\pi$ half-plane. This results from the non-uniform velocity at the cylinder exit at early times (e.g., Fig. 5). The non-uniformity increases with incline angle; thus the initial center of momentum is further from the centerline for larger incline. The center of momentum first moves toward the $\psi=0$ half-plane because of the sweep motion upstream of the primary ring (e.g., Figs. 13(b) and 13(c)). The trajectory then turns toward $\psi=\pi$ due to the induced motion of the vortex structure after it is completely detached from the cylinder (e.g., Figs. 13(d) through 13(f)). Overall, the momentum impulse moves into the $\psi=\pi$ half-plane, and the migration increases with incline angle.

The symbols in Fig. 21 represent identical times for each cylinder case starting at $t^*=1.56$. Thus, we can conclude that the propagation speed of the vortex structure decreases with incline angle. The most extreme comparison can be made between the D cylinder case, shown in Fig. 19(e) at $t^*=4.74$, and the flat cylinder case (Fig. 3(d)). At $t^*=4.74$, the momentum center for the axisymmetric ring is 1.5 times further from the cylinder than in the D case. Over the time interval shown, a representative axial propagation speed for the D case is estimated to be 14.1 mm/s (0.18U_0), significantly slower than for the axisymmetric ring. Note also that in each inclined case, the spacing between data points is non-uniform, indicating that the propagation speed is not constant. This observation is consistent with the evolution of the complex vortex structure.

Finally, it should be noted that the flow field in the D case is already very disorganized by $t^*=4.74$. Beyond this time, the center of momentum in the D case was not calculated because the momentum distribution spread beyond the radial span of the field of view. Flow visualization observations indicate that in this case the propagation of the impulsively driven fluid dies out within a few diameters downstream of the cylinder. This result strongly contrasts with the axisymmetric case, where a coherent ring penetrates steadily to the end of the tank facility ($x/D>8$) with significant momentum. The D/2 and D/4 cases fall between these extremes.

By comparing the flow fields at similar times, it is clear that the fluid motion becomes increasingly disorganized with increasing exit incline angle. This is also clear from the vorticity fields at $t^*=2.62$ and the shapes of the various vortex cores. Even though each case was generated by the same piston impulse, the resulting penetration distance decreased with increasing cylinder incline angle. Flow visualization, the velocity fields, and the center of momentum location illustrate a strong correlation between “coherence” of motion (or coherent circulation) and penetration distance. The loss of coherent circulation caused by inclining the cylinder exit impedes the flow propagation because of reduced induction. This is consistent with the results of Weigand and Gharib for turbulent axisymmetric rings. Furthermore, the degeneration of the perturbed vortex structure into smaller scales and the resulting disorganization must cause increased viscous dissipation, thus further reducing the ability of the initial impulse to penetrate the ambient fluid.

### C. Effects of Re and piston geometry

Tests were performed at Re=23000 (nominally turbulent for L/D=1) in order to evaluate the effect of Reynolds num-
ber on the flow. The axisymmetric ring at this Re propagates at 170 mm/s which corresponds to 2\pi U_v D/\Gamma = 2.9. The ring diameter again decreases slightly after the piston motion stops and then stays constant at 1.13D within the measurement region.

Figure 22 shows the velocity field for the D/2 cylinder and flat piston face at t* = 2.85. The dimensionless time is very similar to the velocity field shown in Fig. 7 for the lower Re (t* = 2.62). A comparison of the figures reveals that the large-scale structure and general features of the flow are Reynolds number independent. The vortex core in the larger Re case is slightly further from the cylinder (which would be expected given the difference in t*), but the size and shape are similar in each half-plane. In addition, in Figs. 7(a) and 22(a) the sweep motion into the \psi = 0 half-plane appears very similar. Finally, in the orthogonal view, the localized region of large axial velocity near the centerline is observed at each Reynolds number.

A series of tests were performed at Re = 23000 to determine the influence of piston geometry on the flow behavior. All previous figures resulted from the case of a flat axisymmetric piston face stopping one diameter upstream of the average cylinder exit location. Additional pistons were manufactured with inclined faces matching the cylinder exit incline. Two arrangements were tested: the inclined piston stopping flush with the cylinder exit, and the inclined piston stopping one diameter upstream. These arrangements yielded qualitatively similar results.

Figure 23 shows the velocity field for the D/2 cylinder with the inclined piston face stopping flush with the exit. Again, the general flow features are qualitatively similar to those observed in the flat piston case (Fig. 22) and the lower Re case (Fig. 7). In particular, the core location, shape, and size, the cross stream sweep, and the centerline region of large axial velocity all have the same general features. It is important to realize (from conservation of mass arguments) that the volume and "stroke length" of displaced fluid is the same for each piston arrangement. Thus, the shear layer growth on the inner wall of the cylinder should be similar regardless of piston geometry, or piston end position. Therefore, any observed differences result from the boundary condition imposed by the piston face on the fluid. For example, the no-slip condition enforced by the piston flush with the cylinder edge yielded relatively small velocities close to the piston in the cross stream sweep (and parallel to the cylinder exit plane). Cases where the piston stopped upstream of the cylinder edge yielded larger sweep velocities close to the cylinder edge plane and some flow into the cylinder cavity during the sweep. The D/4 cylinder case yielded similar independence of the Reynolds number and piston geometry. The D cylinder case was not tested extensively.

**D. Propagation and dispersion of fluid exiting cylinder**

We now consider flow visualization sequences in order to examine additional aspects of the flow behavior. Each image shows an integrated view of the three-dimensional...
flow. The sequences depict the propagation of fluid that initially resided inside of the vortex cylinder. Therefore, detailed behavior of outer entrained fluid is not always observable. In each sequence, the Reynolds number is 23000, and the piston finishes flush with the cylinder exit.

Figure 24 shows a time sequence for flow exiting the flat cylinder. In this case, we note that the front of dark fluid exiting the cylinder is stretched radially outward while ambient fluid is entrained from upstream into the axisymmetric ring that forms. The leading dark fluid is wrapped into the core, while trailing fluid is focused and stretched along the centerline. As the ring propagates forward, a significant amount of dark fluid remains attached to the stopped piston face. This fluid, which becomes associated with an image vortex, rotates and appears to splash radially outward as time progresses. The leading vortex retains a coherent ring of marked fluid all the way to the end of the tank (x/D>8). In the sequence shown, the core and wake appear laminar until x/D=2 (t*=5.75). Beyond this point, small-scale instabilities begin to cause increased shedding of marked core fluid, and the wake appears turbulent. The "turbulent" wake can be observed in the frame taken at t*=8.63.

Figures 25 and 26 show end (normal to the $\psi=\pi/2$ and $3\pi/2$ plane) and side (normal to the $\psi=0$ and $\pi$ plane) views of sequences of flow exiting the D/4 cylinder. In the end (or symmetric) view, the initial entrainment and roll up appear very similar to flow from the axisymmetric reference cylinder, and the core cross section (with transparent center) is clearly visible until t*=4.89. In the side view, we observe a front of dark fluid wrapping into a vortex tube. The trailing marked fluid is swept toward the $\psi=0$ half-plane as outer fluid appears to be entrained from the $\psi=\pi$ half-plane. This is consistent with the sweep motion observed in the velocity fields. The resulting vortex tube is asymmetric; it appears that much more fluid is entrained and wrapped into the $\psi=\pi$ side of the ring as would be expected due to the larger circulation there. As observed in the velocity fields, the ring, which is first inclined at an angle smaller than the cylinder incline, flattens and rotates until eventually the core is tilted in a sense opposite to the cylinder exit. Subsequent frames show substantial shedding of marked fluid into the wake beginning about x/D=2 (t*=5.75). The wake width is fairly similar to that occurring in the flat cylinder case.

Note that a dark volume of trailing fluid remains close to the piston end and propagates outward beyond the cylinder edge before stalling. This volume is analogous to the axisymmetric outward "splash" of marked fluid observed in the flat cylinder case. In the D/4 flow, this trailing fluid is pushed outward into the $\psi=0$ half-plane by the sweep motion. An early stage of this "splash" is observable in Fig. 17(c). In addition, it is clear from the video that the primary vortex ring propagates at an angle to the cylinder axis such that it moves in the $\psi=\pi$ direction, consistent with the trajectory plotted in Fig. 21. The propagation angle was measured as approximately 7.5° when the core was at x/D=3.5 (t*>10). The vortex is very disorganized by x/D=4 and appears to be breaking down. Up to this point, the propagation speed of the D/4 vortex ring is slightly slower than that of the reference vortex, which is again consistent with Fig. 21.

Figures 27 and 28 show the corresponding case of flow exiting the D/2 cylinder. In the end view, it is obvious that the vortex ring is distorted even as it forms. The ring diameter in this view decreases with time as the marked fluid near
the centerline moves ahead of it. By $t^* = 2.01$, the ring diameter is already smaller than in the D/4 or flat cases, and by $t^* = 3.45$, a distinct core is no longer observable. In this view, we observe a significant amount of marked core fluid shed into the wake beginning near $t^* = 6.33$. The wake in this view is slightly broader than that in the flat or D/4 cases.

In the side view of the D/2 case, we see the roll up of a strong vortical structure on the $\psi = \pi$ side. On the $\psi = 0$ side, however, there is no evidence of any core or roll up of fluid even though such a core is clearly present in the velocity fields (Fig. 13). We believe this core is not observed in the visualization because the leading fluid exiting the tip of this

![Flow visualization images](image-url)
more-inclined cylinder was not darkened by our pre-charging method. In addition, a substantial part of the fluid within this core must be entrained from outside (see Fig. 5). A sweep of entrained fluid moving from the \( \psi = \pi \) side toward the \( \psi = 0 \) side trails the leading marked fluid. At first, the marked fluid remains attached to the cylinder trailing edge at \( \psi = 0 \), but then it is swept outward and downstream. This forms part of a broad strip of marked fluid which remains attached to a large diffuse core on the \( \psi = \pi \) side (\( t^* = 5.18 \)). At later times, the marked fluid appears as two separate volumes with the \( \psi = \pi \) side leading. It is difficult to discern any coherent vortex core from the flow visualization at these later times. In this view, the wake structure, which appears highly disorganized, is broader than in the flat and D/4 cases. In addition,
the propagation speed of the D/2 structure is clearly smaller than that of either the flat or D/4 rings, and the penetration appears to die out altogether by x/D=4 (not shown in figures).

IV. SUMMARY AND CONCLUSIONS

A simple geometric perturbation to a typical vortex ring generator has a strong impact on the flow dynamics. Inclining the cylinder exit significantly alters both the vortex ring formation and the subsequent flow evolution. The initial path lines exiting the cylinder curve toward the $\psi=\pi$ half-plane (shortest cylinder location), and the velocity magnitude exiting the orifice varies spatially, resulting in a primary vortex ring with non-uniform circulation. A complex structure forms, consisting of the primary vortex ring and branched vortex tubes extending upstream toward the cylinder boundary. The circulation non-uniformity in the primary ring and the strength of the branched vortex tubes increase with exit incline angle. The branched tubes initially induce a cross-stream sweep motion upstream of the primary ring.

Around the time that the sweep collides with the vortex core in the $\psi=0$ half-plane (longest cylinder location), the branched vortex tube breaks free of the cylinder and propagates through the primary ring. Beyond this time, the vortex structure consists of the primary ring and a leading closed-loop branched tube. The induced motion of the structure moves the center of momentum toward the $\psi=\pi$ half-plane (shortest cylinder location). As the incline angle increases, the extent of the migration away from the centerline, the disorganization of the fluid motion, and the width of the wake increase. The earlier loss of coherent circulation leads to decreased propagation speed and penetration distance. The general features of the flow are independent of the Reynolds number and piston geometry.

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